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Equation of a Particle in Gravitational Field of Spherical Body

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Abstract

In this study the law of classical mechanics for the corpuscular behavior of all entities in all interaction fields in nature is formulated. This law is applied to an entity of non-zero rest mass in the gravitational field of stationary homogeneous spherical body. The results are that the classical mechanical equation hence the solutions of motion for the entity contains terms of all orders of c^{-2}, the solution of motion for the entity predicts an anomalous orbital precession in perfect agreement with experiment and Einstein theory of general relativity.

Keywords: Non-zero mass, Gravitational field, Relativity and Inertial mass

1. Introduction

Usually, we talk rather glibly about the masses of body that describes quite different properties. They are;

i. **INERTIAL MASS** f m^i, which is a measure of the body's resistance to change in motion
ii. **MASSIVE GRAVITATIONAL MASS** m^p, which is a measure of its reaction to gravitational field.
iii. **ACTIVE GRAVITATIONAL MASS** m^a, which is a measure of its source strength from producing a gravitational field.

In this study, we present our definition of general mass as "Based upon the experimental data available today, we assert as follows, to the degree of accuracy of measurement available, the complete instantaneous mass m associated with an entity is its complete instantaneous classical non-potential energy k divided by the square of the speed of light in vacuo c"

$$m = \frac{k}{c^2} \tag{1}$$

This assertaion is our definition of mass

It is always established to the degree of accuracy of measurement available today, that if an entity of non- zero rest mass m is moving with an instantaneous classical non-potential energy k is given by

$$k = \left[1 - \frac{u^2}{c^2}\right] m_0 c^2 \tag{2}$$

It follows from the definition that our instantaneous mass m for an entity of non-zero rest mass m_0 moving with instantaneous velocity u is given by

$$k = \left[1 - \frac{u^2}{c^2}\right]^{1/2} m_0 \tag{3}$$

Finally, from our equivalent principle and our philosophy of classical mechanics, we assert as a most natural extension of generalization of Newton's law of classical mechanics, as follows: "In all interaction fields in nature the time rate of change of our instantaneous linear momentum $\vec{p}$ of an entity equal to the total instantaneous external force F^e acting on it:

$$\frac{d}{dt}\vec{p} = \vec{F}^e \text{ "} \tag{4}$$

This assertion is our law of classical mechanics.

This result completes for practice, our theory of classical mechanics for entities of non-zero rest masses.

2. Mechanical Equations of Motion

Consider an entity of non-zero rest mass m_0 moving in the gravitational field of a stationary homogeneous spherical body of radius R and rest mass M_0. Let S be the reference frame whose origin O coincides with the centre of the body. Then from the law of classical gravitation, it follows that our instantaneous classical gravitational force $\vec{F}_g$ acting on the entity is given in the spherical co ordinates

$\vec{r}: \vec{r}(r, \phi, \theta)$

of S as

$$\vec{F}_g(\vec{r}.\, t) = -\frac{mk}{r^2}\hat{r} \tag{5}$$

where

$$K=GM \tag{6}$$

where G is the universal gravitational constant. Hence by our law of classical mechanics our classical mechanical equation of motion for the entity is given by

$$\frac{d}{dt}\left[m_0\left(1-\frac{u^2}{c^2}\right)^{1/2}\vec{u}\right] = -\frac{mk}{r^2}\hat{r} \tag{7}$$

where $\vec{u}$ is the instantaneous velocity.

Our equation of motion may be written as equivalent and more conveniently in the form,

$$-\frac{k}{r^2}\hat{r} = \frac{d}{dt}\vec{u}\left[\frac{1}{2c^2}\left(1-\frac{u^2}{c^2}\right)^{-1}\frac{d}{dt}u^2\right]\vec{u} \tag{8}$$

but in spherical coordinates,

$$\vec{u} = \dot{r}_g + r\dot{\theta}_{\hat{\theta}} + r\dot{\phi}sin\theta_{\hat{\phi}} \tag{9}$$

and

$$\frac{d\vec{u}}{dt} = (\ddot{r} - r\dot{\theta}sin\theta - r\phi sin^2\theta)\hat{r} + (r\phi sin\theta + 2\ddot{r}osin\phi + 2\phi cos\theta)\phi \tag{10}$$

Hence the r-component of our classical mechanical equation of motion for the entity is given by

$$-\frac{k}{r^2} = \ddot{r} - r\dot{\theta}^2 sin\theta - r\dot{\phi}^2 + \frac{r}{2c^2}\left(1-\frac{u^2}{c^2}\right)^{-1}\frac{d}{dt}u^2 \tag{11}$$

and the corresponding θ component is given by

$$0 = r\ddot{\theta} + 2\dot{r}\dot{\theta} - r\dot{\theta}^2 sin\theta cos\theta + \frac{r\theta}{2c^2}\left(1-\frac{u^2}{c^2}\right)^{-1}\frac{d}{dt}u^2 \tag{12}$$

and the corresponding φ component is given by

$$0 = r\,\ddot{\phi}sin\theta + 2\dot{r}\dot{\phi}sin\theta + 2\dot{r}\dot{\theta}\dot{\phi}cos\theta + \frac{r\dot{\phi}sin\theta}{2c^2}\left(1-\frac{u^2}{c^2}\right)^{-1}\frac{d}{dt}u^2 \tag{13}$$

Equations (11), (12), and (13) are our general equations of motion in their various components. For the sake of mathematical convenience, let the entity move in the equatorial plane of the body $\theta = \frac{\pi}{2}$

Then the general mechanical equations (11-13) reduce to

$$-\frac{k}{r^2} = \ddot{r} - r\dot{\phi}^2 + \frac{\dot{r}}{2c^2}\left(1-\frac{u^2}{c^2}\right)^{-1}\frac{d}{dt}u^2 \tag{14}$$

And

$$0 = r\ddot{\phi} + 2\dot{r}\dot{\phi} + \frac{r\dot{\theta}}{2c^2}\left(1 - \frac{u^2}{c^2}\right)^{-1}\frac{d}{dt}u^2 \tag{15}$$

Where

$$u^2 = \dot{r}^2 + r^2\dot{\phi}^2 \tag{16}$$

Equations (14) and (15) are our equations of motion of the entity in the equatorial plane.

3. Angular Speed in the Equatorial Plane

The mechanical angular equation for the entity in the equatorial plane (15) may be written as

$$0 = \frac{\ddot{\phi}}{\dot{\phi}} + \frac{2\dot{r}}{r} + \frac{1}{2c^2}\left(1 - \frac{u^2}{c^2}\right)^{-1}\frac{d}{dt}u^2 \tag{17}$$

Which integrates exactly to give

$$\dot{\phi} = \frac{\ell}{r^2}\left(1 - \frac{u^2}{c^2}\right)^{1/2} \tag{18}$$

Where l is the constant of motion.

Substituting the expression for u^2 from (16) we have

$$\dot{\phi} = \frac{\ell}{r^2}\left(1 - \frac{\dot{r}^2}{c^2}\right)^{1^1/2}\left(1 + \frac{\ell^2}{r^2c^2}\right)^{-1/2} \tag{19}$$

This is our full classical mechanical angular speed of the entity in the equatorial plane. And it follows that the constant l is precisely our classical mechanical angular momentum per unit mass of the entity.

It may be noted that to the zero order of c^{-2} our classical mechanical angular speed of the entity in the equatorial plane reduces to

$$\dot{\phi} = \frac{\ell}{r^2} \tag{20}$$

which is precisely Newton's classical mechanical angular speed of the entity in the equatorial plane.

4. Radial Equation in the Equatorial Plane

The classical mechanical radial equation of motion for the entity in the equatorial plane (14) may be written as

$$\ddot{r} + \frac{k}{r^2}\phi\sin\theta - r\dot{\phi}^2 = r\frac{d}{dt}\left[\ln\left(1 + \frac{u^2}{c^2}\right)\right]^{1/2} \tag{21}$$

Substituting the expression (16) for u^2 followed by our full classical angular speed (19), it follows that

$$= \dot{r}\frac{d}{dt}\left\{\ln\left[1 - \frac{1}{c^2}\left\{\dot{r}^2 + \frac{\ell^2}{r^2}\left(1 - \frac{\dot{r}^2}{c^2}\right)\left(1 + \frac{\ell^2}{c^2r^2}\right)^{-1}\right\}\right]\right\} \tag{22}$$

This is the full classical mechanical radial equation of motion for the entity in the equatorial plane.

Note that to the zero order of c^{-2} the full classical mechanical radial equation motion for the entity in the equatorial plane reduces to

$$\ddot{r} + \frac{k}{r^2} - \frac{\ell^2}{r^3} = 0 \tag{23}$$

Which is precisely Newton's classical mechanical radial equation of motion for the entity.

To the first order of c^{-2}

Equation (22) reduces to

$$\left(1-\frac{\dot{r}^2}{c^2}\ddot{r}=\frac{k}{r^2}-\frac{\ell^2}{r^3}\right) \tag{24}$$

by the transformation

$$\mathrm{W(r)}=\dot{r}(r) \tag{25}$$

It follows that (24) integrates exactly to yield (Howusu, 1993)

$$1+\frac{\dot{r}^2}{c^2}=\left\{1+\frac{4}{c^2}\left[k\left(\frac{1}{r}-\frac{1}{r_i}\right)-\frac{\ell^2}{2}\left(\frac{1}{r^2}-\frac{1}{r_i{}^2}\right)\right]\right\} \tag{26}$$

Where r_i is any apsidal distance. This result specifies our classical mechanical radial speed of the entity corresponding to our classical mechanical radial equation (24).

To the first order of c^{-2} our classical mechanical radial speed (26) reduces to

$$\dot{r}=2\left[k\left(\frac{1}{r}-\frac{1}{r_i}\right)-\frac{\ell^2}{2}\left(\frac{1}{r^2}-\frac{1}{r_i{}^2}\right)\right]$$
$$-\frac{2}{c^2}\left[k\left(\frac{1}{r}-\frac{1}{r_i}\right)-\frac{\ell^2}{2}\left(\frac{1}{r^2}-\frac{1}{r_i{}^2}\right)\right]^2 \tag{27}$$

Now by substituting (26) into (24) it follows that

$$\ddot{r}=\left[-\frac{k}{r^2}+\frac{\ell^2}{r^3}\right]\left\{1+\frac{4}{c^2}\left[k\left(\frac{1}{r}-\frac{1}{r_i}\right)-\frac{\ell^2}{2}\left(\frac{1}{r^2}-\frac{1}{r_i{}^2}\right)\right]\right\} \tag{28}$$

Where r_i is any apsidal distance. This is our classical mechanical radial acceleration of the entity corresponding to our classical mechanical equation (24). To the first order of c^{-2} our classical mechanical radial acceleration of the entity (28) reduces to

$$\ddot{r}=-\frac{k}{r^2}\left[1+\frac{1}{c^2}\left(\frac{2k}{r_i}-\frac{\ell^2}{r_i{}^2}\right)\right]+\frac{\ell^2}{r^3}\left[1+\frac{1}{c^2}\left(\frac{2k}{r_i}-\frac{\ell^2}{r_i{}^2}+\frac{2k^2}{\ell^2}\right)\right]-\frac{3k\ell^2}{r^4}+\frac{\ell^4}{c^2r^5} \tag{29}$$

The equation (24) and (26) are sufficient solutions of our classical mechanical radial equation of the first order of c^{-2}, (24) in terms of the radial coordinate. It is, however, instructive to express the radial equation and hence its solution in terms of the angular co-ordinate. Hence, let V be a new variable defined by

$$v(\phi)=\frac{1}{r(\phi)} \tag{30}$$

Then, it follows from the zero order of c^{-2}, approximation (20) of our full classical mechanical angular speed of the entity, that our first order of c^{-2} radial equation (29) transforms as

$$\frac{k}{\ell^2}\left[1+\frac{1}{c^2}\left(\frac{2k}{r_i}-\frac{\ell^2}{r_i{}^2}\right)\right]=\frac{d^2V}{d\phi^2}$$
$$+\left[1+\frac{1}{c^2}\left(\frac{2k}{r_i}-\frac{\ell^2}{r_i{}^2}+\frac{2k^2}{c^2}\right)\right]V-\frac{3k}{c^2}v^2+\frac{\ell^2}{c^2}v^3 \tag{31}$$

Let our body be the sum (assumed to be homogeneous and stationary). Let the entity be a planet or an asteroid. Then from the astronomical data for the solar system (Moore, 1980)

R=6.96 x 10^8 m

M=1.989 x 10^{30} kg

$r_i=10^{10}$ m

L=10^{30} m^2s^{-1}

Hence, G=6.672 x 10^{-11} Nm^2kg^{-2} our classical mechanical radial equation for the entity (31) may be approximated by

$$\frac{k}{\ell^2} = \frac{d^2V}{d\phi^2} + v - \frac{3k}{c^2}v^2 \tag{32}$$

This equation is precisely the same as the well known Eintein's equation for the motion of the planets and asteroids in the solar system from general relativity. And an angle θ is given by

$$\theta = \frac{6\pi GM}{a(1-e^2)c^2} \tag{33}$$

where a is the semi-major axis and e is the eccentricity of the orbit. This expression is well known to be in excellent agreement with astronomical measurements for the solar system.

5. Conclusion

A derivation of our classical mechanical radial equation in the equatorial plane to all orders of c^{-2} in terms of the radial coordinates is now obvious. The solutions reveal our classical mechanical corrections of all orders of c^{-2} to the solution corresponding to Newton's classical mechanical radial equation.

Our theory of classical mechanics has resolved the famous gravitational problem of anomalous orbit precession of bodies in the solar system. Consequently, Einstein's theory of general relativity is not necessary for the resolution of this problem as hitherto assumed.

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